The Mathematical Path

Introduction to Factoring Polynomials–Common Factor

Factoring is the process of taking an expression that is a sum of products and converting it into a product of sums.

This module introduces the concept of factoring and works through some examples of factoring out common factors from polynomials.

This module assumes you're familiar with:

- Factoring Integers
- Introduction to Exponents
- Introduction to Polynomials

If you are already familiar with expanding polynomials, you will find connections with that work, but it's not essential to understand this material.

This module is a part of The Mathematical Path, which introduces mathematical ideas step-by-step to ensure that everyone's able to follow along. The latest version of the file can be obtained from `www.mathematicalpath.com/` along with other supporting materials.

Remember that mathematics builds upon itself, so if you're struggling with this material, it could be because you didn't quite understand something earlier on. Don't be afraid to back up and review some earlier mathematical concepts to get a stronger foundation and then come back to this module when you're ready. Visit The Mathematical Path website to trace back until you reach material you're comfortable with and then work your way back up to this module.

Be sure to visit `www.mathematicalpath.com/support/` to learn how you can support The Mathematical Path so we can create more modules!

A lot of the operations we do in mathematics come in pairs that are the opposites of each other. Adding and subtracting are one example of this. Multiplying and dividing are another example. This module introduces the idea of factoring, which is the opposite of expanding. (In some places, "factoring" is called "factorisation", but we will use the term "factoring" throughout this module.)

Throughout this module and in subsequent ones dealing with factoring, we will be making changes to the presentation of expressions without actually changing the values. For example, multiplying an expression by 1 or adding 0 to it does not change the overall value. However, in doing so, we'll change the presentation of the expression. This can make it easier to see properties of an expression.

There are two common ways of representing algebraic expressions. One way is as a sum of products which look like this:

$$(\_\_ \times \_\_ \times \ldots \times \_\_) + (\_\_ \times \_\_ \times \ldots \times \_\_) + \ldots + (\_\_ \times \_\_ \times \ldots \times \_\_)$$

That is, there are a series of terms, each of which is a product, that are added together.

Another common way of representing algebraic expression is as a product of sums, which look like this:

$$(\_\_ + \_\_ + \ldots + \_\_) \times (\_\_ + \_\_ + \ldots + \_\_) \times \ldots + (\_\_ + \_\_ + \ldots + \_\_)$$

That is, there a series of terms, each of which is a sum, that are multiplied together.

Factoring is a process of converting an expression that is a sum of products into a product of sums. This is the opposite of expanding (which is the process of converting a product of sums into a sum of products). One nice thing about this relationship is that you can always check your factoring result by expanding it again. If you don't get the same thing you started with, you know something went wrong along the way.

One note before we dive in: When we talk about sums, it actually doesn't matter if some of those additions are actually subtractions! Remember that subtraction is the same as adding a negative number. We'll have a few examples of this as we go.

Finding a constant factor

Let's start by focusing on the numbers or coefficients. We are going to look at all the terms in the expression and see if there are any numbers that evenly divide all them. Since we want to factor expressions as much as possible, we'll choose the largest number that factors all the coefficients.

We begin by ensuring we can properly identify the coefficient values in polynomials.

Problem set 1 (identifying coefficients):

For each polynomial, write a list of all the coefficients of the terms.

Sample problem: $2x^2 + 4x - 6$

Sample solution: Because this expression includes a subtraction, we'll change it so that we're adding a negative number to make the coefficients clearer. We can re-write this equation as: $2x^2 + 4x + (-6)$

The coefficient on the x^2 term is 2. The coefficient on the x term is 4. The coefficient on the term without an variable is -6.

The list of coefficients is: 2, 4, and -6

a) $5x^2 + 2x - 8$

b) $3x^2 + x + 4$

c) $8x^2 + 5y^2 + 9x + y + 4$

d) $-2x - 5y - 10$

Once we have a list of coefficients, we find the factors of each of them. The largest factor which is shared by all the coefficients is the one we will use for factoring.

Problem set 2 (identifying largest numeric factors):

For each algebraic expression, find the list of coefficients. For each coefficient, list all the positive integer factors (numbers that are greater than zero and divide the coefficient evenly without leaving a remainder). Find the largest factor that is shared by all the coefficients.

Sample problem: $2x^2 + 4x - 6$

Sample solution: From the previous problem set, we know the coefficients are: 2, 4, and -6.

The positive integer factors of 2 are: 1 and 2. The positive integer factors of 4 are: 1, 2, and 4. The positive integer factors of -6 are: 1, 2, 3, and 6.

The largest factor that is shared by all the coefficients is 2.

a) $3x + 9y + 3$

b) $10x - 6y + 8$

c) $24x^2 + 6x - 18y + 12$

d) $6x + 12y + 5$

We can multiply the expression by 1 without changing its value. So, we multiply the expression by $1 = \frac{factor}{factor}$. We can then re-arrange the equation in order to get the factored expression, distributing the denominator across the original expression. A sample problem will make this process clearer.

Problem set 3 (factoring the numeric portion):

For each algebraic expression (repeated from the previous problem set), factor out the largest numeric factor possible.

Sample problem: $2x^2 + 4x - 6$

Sample solution: From the previous problem set, we know the largest factor that is shared by all the coefficients is 2.

We multiply this expression by $1 = \frac{2}{2}$. This does not change the value of the expression.

$$\frac{2}{2}(2x^2 + 4x - 6)$$

We can rearrange this expression, splitting the fraction into two pieces, again without changing the value.

$$\frac{2}{1}\frac{1}{2}(2x^2 + 4x - 6)$$

Next, we can expand the $\frac{1}{2}$ fraction into the polynomial, multiplying it by each of the terms.

$$\frac{2}{1}\left(\frac{2x^2}{2} + \frac{4x}{2} - \frac{6}{2}\right)$$

We can then simplify these fractions.

$$2(1x^2 + 2x - 3)$$

As a final clean-up step, we remove the unnecessary coefficient 1 on the x^2 term because it would be implied.

$$2(x^2 + 2x - 3)$$

a) $3x + 9y + 3$

b) $10x - 6y + 8$

c) $24x^2 + 6x - 18y + 12$

d) $6x + 12y + 5$

So far we have focused on factoring only positive integers. However, we can equally factor out a negative number. Sometimes this is a stylistic choice–it is often easier to read expressions that have fewer negative terms in them. Other times, we may prefer to factor a specific term or have a specific result remaining after factoring, impacting our selection of a positive or negative factor.

Problem set 4 (factoring negative terms):
For each algebraic expression, factor out the positive and negative versions of the numeric factor with the largest magnitude.

Sample problem: $10x - 5y$

Sample solution: The coefficients are 10 and -5 and the largest shared positive factor is 5. We can factor out 5:

$$10x - 5y = \frac{5}{5}(10x - 5y) = 5\left(\frac{10x}{5} + \frac{-5y}{5}\right) = 5(2x - y)$$

Alternatively, we can factor out -5:

$$10x - 5y = \frac{-5}{-5}(10x - 5y) = -5\left(\frac{10x}{-5} + \frac{-5y}{-5}\right) = -5(-2x + y) = -5(y - 2x)$$

a) $-2x^2 - 6y$

b) $9z - 15$

Finding a variable factor

Just as we can factor out numbers from the coefficients, we can similarly factor out variables from the terms.

Recall that exponents are just a short-hand way of multiplying. This means that $x^3y^2z = xxxyyz$.

Just as we considered the numeric coefficient separately from the algebraic variables, we can consider each of the different variables separately too. If we are factoring out x terms from x^3y^2z, we can factor out a single x, two x variables (x^2), or all three x variables (x^3). We can also factor out zero x values, which is equivalent to factoring out $x^0 = 1$. That is, we can factor out any number of copies of a variable from 0 up to the exponent of that variable.

Problem set 5 (identifying variable factors):
For each product there is a list of possible factors. Identify all the possible factors that are actually factors of the product.

Sample problem: x^4y^2: x^2y xyz y x^2y^4

Sample solution: We will consider each potential factor separately.

x^2y: The exponent on the x term in the potential factor is 2, which is less than the exponent 4 on the x term in the product. The exponent on the y term in the potential factor is 1 (which is not displayed, but is implied), which is less than the exponent 2 on the y term in the product. Therefore x^2y is a factor of x^4y^2.

xyz: The exponent on the x term in the potential factor is 1 (which is not displayed, but is implied), which is less than the exponent 4 on the x term in the product. The exponent on the y term in the potential factor is also 1, which is less than the exponent 2 on the y term in the product. The potential factor has a z term (with an implied exponent of 1), but there is no z term in the product. This is the same as the potential factor having a z term with an exponent of 0 (because $z^0 = 1$). The exponent on the z term is larger in the potential factor on the z term in the product. Therefore, xyz is not a factor of x^4y^2 (because of the z term).

y: There is no x term in the potential factor. This is the same as there being a x term with an exponent of 0 (because $x^0 = 1$). This is less than the exponent 4 on the x term in the product. The exponent on the y term in the potential factor is 1 (which is not displayed, but is implied), which is less than the exponent 2 on the y term in the product. Therefore, y is a factor of x^4y^2.

x^2y^4: The exponent on the x term in the potential factor is 2, which is less than the exponent 4 on the x term in the product. The exponent on the y term in the potential factor is 4, which is greater than the exponent 2 on the y term in the product. Therefore x^2y^4 is not a factor of x^4y^2 (because of the y term).

In summary, x^2y and y are factors of x^4y^2, but xyz and x^2y^4 are not.

a) x^5y^3z: xz^2 xyz x^4y^4 ax^2y

b) yz^2: x yz y^2z 1

c) xyz: xy xz x^2 q

d) $x^2y^5z^3$: x^3y^2 $x^2y^3z^2$ y^3x^2 $x^2y^5z^3$

To factor variables from an algebraic expression, we want to find the largest combination of variables that are a factor of all the terms.

Problem set 6 (identifying largest variable factors):

For each algebraic expression, find the largest variable factor that is shared by all the terms.

Sample problem: $2x^2y^3z^4 - 5x^3y + 6x^5y^4z$

Sample solution: We consider each of the variables separately.

The exponents on the x variable are 2, 3, and 5 respectively. The smallest exponent is 2. Therefore, x^2 can be factored out of all the terms.

The exponents on the y variable are 3, 1 (not displayed, but implied), and 4. The smallest exponent is 1 Therefore, $y^1 = y$ can be factored out of all the terms.

Not all the terms have a z variable. If a term doesn't have a z factor, this is equivalent to $z^0 = 1$ with an exponent 0. Therefore, the exponents on the z variable are 4, 0, and 1 respectively. The smallest exponent is 0. Therefore, $z^0 = 1$ can be factored out of all the terms, which is the same as no z variables being factored out.

Putting these together, the largest shared factor is x^2y.

a) $9x^5y^2z^3 + 3x^3y^7z^2 - 2x^2y^6z^{10}$

b) $5x^6y^3z^2 - 6x^9z^6 + 2x^5y^2z^5$

c) $10x^{12}y^3 + 5x^8z^8 + 6y^9z^4$

d) $3x^9y^5z^7 - 2x^6y^8z^{10} - 9x^{13}y^8z^9$

Just as before, we can multiply the expression by 1 without changing its value. So, we multiply the expression by $1 = \frac{factor}{factor}$, but in this case the factor is comprised of variables. We can then rearrange the equation in order to get the factored expression, distributing the denominator across the original expression.

Problem set 7 (factoring the variable portion):

For each algebraic expression (repeated from the previous problem set), factor out the largest variable factor possible.

Sample problem: $2x^2y^3z^4 - 5x^3y + 6x^5y^4z$

Sample solution: From the previous problem set, we know the largest variable factor that is shared by all the terms is x^2y.

We multiply this expression by $1 = \frac{x^2y}{x^2y}$. This does not change the value of the expression.

$$\frac{x^2y}{x^2y}(2x^2y^3z^4 - 5x^3y + 6x^5y^4z)$$

We can rearrange this expression, splitting the fraction into two pieces, again without changing the value.

$$\frac{x^2y}{1}\frac{1}{x^2y}(2x^2y^3z^4 - 5x^3y + 6x^5y^4z)$$

Next, we can expand the $\frac{1}{2}$ fraction into the polynomial, multiplying it by each of the terms.

$$\frac{x^2y}{1}\left(\frac{2x^2y^3z^4}{x^2y} - \frac{5x^3y}{x^2y} + \frac{6x^5y^4z}{x^2y}\right)$$

We can then simplify these fractions.

$$x^2y(2x^0y^2z^4 - 5x^1y^0 + 6x^3y^3z)$$

As a final clean-up step, we remove the unnecessary 1 exponents (it is implied) and remove the variables with exponents equal to 0 (because variables to the exponent 0 are equal to 1).

$$x^2y(2y^2z^4 - 5x + 6x^3y^3z)$$

a) $9x^5y^2z^3 + 3x^3y^7z^2 - 2x^2y^6z^{10}$

b) $5x^6y^3z^2 - 6x^9z^6 + 2x^5y^2z^5$

c) $10x^{12}y^3 + 5x^8z^8 + 6y^9z^4$

d) $3x^9y^5z^7 - 2x^6y^8z^{10} - 9x^{13}y^8z^9$

Finding the greatest common factor

We can now combine these ideas to find the greatest common factor which is a combination of both the numeric and variable factors. The greatest common factor is the largest possible values that can be factored out from a set of terms.

Problem set 8 (identifying the greatest common factor):
For each algebraic expression, find the greatest common factor of the terms. Then factor this greatest common factor out of the expression.

Sample problem: $3x^2y^3 + 6xy^5 - 15x^3y^4z^2$

Sample solution: We will consider the coefficients first, then each of the variables in turn.

The coefficients are: 3, 6, and -15. The positive integer factors of 3 are: 1 and 3. The positive integer factors of 6 are: $1, 2, 3$, and 6. The positive integer factors of -15 are: $1, 3, 5$, and 15. The largest shared factor is 3.

The exponents on the x variable are 2, 1 (implied), and 3 respectively. The smallest exponent is 1. Therefore, $x^1 = x$ can be factored out of all the terms.

The exponents on the y variable are 3, 5, and 4 respectively. The smallest exponent is 3. Therefore, y^3 can be factored out of all the terms.

The exponents on the z variable are 0, 0, and 2 respectively. The smallest exponent is 0. Therefore, $z^0 = 1$ can be factored out of all the terms.

Putting these together, the greatest common factor is $3xy^3$.

Multiply the expression by $1 = \frac{3xy^3}{3xy^3}$, then re-distribute the fraction.

$$\frac{3xy^3}{3xy^3}(3x^2y^3 + 6xy^5 - 15x^3y^4z^2) = 3xy^3\left(\frac{3x^2y^3}{3xy^3} + \frac{6xy^5}{3xy^3} - \frac{15x^3y^4z^2}{3xy^3}\right)$$

We can then simplify these fractions:

$$3xy^3(1x^1y^0 + 2x^0y^2 - 5x^2y^1z^2)$$

As a final step, clean up the expression:

$$3xy^3(x + 2y^2 - 5x^2yz^2)$$

a) $4x^5y^3z^6 - 8x^3y^5z^2 + 6x^4y^3z^7$

b) $5xyz^3 + 10x^2yz^5$

c) $8x^3y^2 - 12x^2$

d) $7x + 8y^2$

Key Points

- Factoring converts a sum of products into a product of sums.
- Operations only change the presentation of the expression, not its value.
- The greatest common factor is the largest value that evenly divides all the terms of the expression.
- Factoring out a common factor from an expression results in that factor multiplied by the original expression but with each term divided by that factor.

Next Steps

If you've read through this module, make sure you do all the exercises and check the solutions in the next section. If you got any answers wrong, go over the solutions and try to understand why you got a different answer.

If you are still feeling confused or don't understand something, please let us know. We're always willing to revisit and improve modules to ensure that everyone is able to follow along. It's not your fault for not understanding something, it's our fault it wasn't presented clearly enough! The best way for us to improve in order to help you and the next person who comes along is for you to tell us what didn't make sense. Similarly, if there's something you thought should be covered in this module but wasn't, be sure to let us know.

If you're feeling confident in this material, check out the `www.mathematicalpath.com/` to find more modules that build on this one. If there's a module you want but doesn't exist yet, contact us to make a request and help us prioritize our efforts. We hope to see you soon as you continue down the Mathematical Path.

Solutions

Solutions to problem set 1

a) This expression includes a subtraction, so convert it adding a negative number to make the coefficients clearer.

$$5x^2 + 2x + (-8)$$

The coefficient on the x^2 term is 5. The coefficient on the x term is 2. The coefficient on the term without an algebraic variable is -8.

The list of coefficients is: 5, 2, and -8.

b) There are no negative terms. The coefficient on the x^2 term is 3. The coefficient on the x term is 1 (even though it isn't written out explicitly in the expression). The coefficient on the term without an algebraic variable is 4.

The list of coefficients is: 3, 1, and 4.

c) There are no negative terms. The coefficient on the x^2 term is 8. The coefficient on the y^2 term is 5. The coefficient on the x term is 9. The coefficient on the y term is 1 (even though it isn't written out explicitly in the expression). The coefficient on the term without an algebraic variable is 4.

The list of coefficients is: 8, 5, 9, 1, and 4.

d) This expression includes a subtraction, so convert it adding a negative number to make the coefficients clearer.

$$-2x + (-5y) + (-10)$$

The coefficient on the x term is -2. The coefficient on the x term is -5. The coefficient on the term without an algebraic variable is -10.

The list of coefficients is: -2, -5, and -10

Solutions to problem set 2

a) The coefficients are: $3, 9$, and 3 again. (We only need to factor 3 once.)

The positive integer factors of 3 are: 1 and 3. The positive integer factors of 9 are: $1, 3$, and 9.

The largest factor that is shared by all the coefficients is 3.

b) The coefficients are: 10, -6, and 8.

The positive integer factors of 10 are: 1, 2, 5, and 10. The positive integer factors of -6 are: 1, 2, and 3. The positive integer factors of 3 are: 1 and 3.

The largest factor that is shared by all the coefficients is 2.

c) The coefficients are: 24, 6, -18, and 12.

The positive integer factors of 24 are: 1, 2, 3, 4, 6, 8, 12, and 24. The positive integer factors of 6 are: 1, 2, 3, and 6. The positive integer factors of -18 are: 1, 2, 3, 6, 9, and 18. The positive integer factors of 12 are: 1, 2, 3, 4, 6, and 12.

There are multiple shared factors, but the largest factor that is shared by all the coefficients is 6.

d) The coefficients are: 6, 12, and 5.

The positive integer factors of 6 are: 1, 2, 3, and 6. The positive integer factors of 12 are: 1, 2, 3, 4, 6, and 12. The positive integer factors of 5 are: 1 and 5.

The largest factor that is shared by all the coefficients is 1.

Solutions to problem set 3

a) From the previous problem set, the largest factor that is shared by all the coefficients is 3.

Multiply the expression by $\frac{3}{3}$, then rearrange the expression:

$$\frac{3}{3}(3x + 9y + 3) = 3\left(\frac{3}{3}x + \frac{9}{3}y + \frac{3}{3}\right)$$

Then simplify the expression:

$$3(x + 3y + 1)$$

b) From the previous problem set, we know the largest factor that is shared by all the coefficients is 2.

Multiply the expression by $\frac{2}{2}$, then rearrange the expression:

$$\frac{2}{2}(10x - 6y + 8) = 2\left(\frac{10}{2}x - \frac{6}{2}y + \frac{8}{2}\right)$$

Then simplify the expression:

$$2(5x - 3y + 4)$$

c) From the previous problem set, we know the largest factor that is shared by all the coefficients is 6.

Multiply the expression by $\frac{6}{6}$, then rearrange the expression:

$$\frac{6}{6}(24x^2 + 6x - 18y + 12) = 6\left(\frac{24}{6}x^2 + \frac{6}{6}x - \frac{18}{6}y + \frac{12}{6}\right)$$

Then simplify the expression:

$$6(4x^2 + 1x - 3y + 2)$$

We can tidy this up because we don't need to write the 1 coefficient on the x term:

$$6(4x^2 + x - 3y + 2)$$

d) From the previous problem set, the largest factor that is shared by all the coefficients is 1.

You may notice immediately that factoring out a 1 will not change an expression. However, we can still go through the process of factoring to confirm this.

Multiply the expression by $\frac{1}{1}$, then rearrange the expression:

$$\frac{1}{1}(6x + 12y + 5) = 1\left(\frac{6}{1}x + \frac{12}{1}y + \frac{5}{1}\right)$$

Then simplify this expression:

$$1(6x + 12y + 5)$$

Multiplying 1 by anything does not change the expression, so this is equivalent to:

$$6x + 12y + 5$$

Since this is the same expression we started with, we cannot factor out a common factor from this expression.

Solutions to problem set 4

a) The coefficients are -2 and -6 and the largest shared positive factor is 2. We can factor out 2:

$$-2x^2 - 6y = \frac{2}{2}(-2x^2 - 6y) = 2\left(\frac{-2x^2}{2} + \frac{-6y}{2}\right) = 2(-x^2 - 3y)$$

Alternatively, we can factor out -2:

$$-2x^2 - 6y = \frac{-2}{-2}(-2x^2 - 6y) = -2\left(\frac{-2x^2}{-2} + \frac{-6y}{-2}\right) = -2(x^2 + 3y)$$

b) The coefficients are 9 and -15 and the largest shared positive factor is 3. We can factor out 3:

$$9z - 15 = \frac{3}{3}(9z - 15) = 3\left(\frac{9z}{3} + \frac{-15}{3}\right) = 3(3z - 5)$$

Alternatively, we can factor out -3:

$$9z - 15 = \frac{-3}{-3}(9z - 15) = -3\left(\frac{9z}{-3} + \frac{-15}{-3}\right) = -3(-3z + 5) = -3(5 - 3z)$$

Solutions to problem set 5

a) We will consider each potential factor separately.

xz^2: The exponent on the x term in the potential factor is 1 (which is not displayed, but is implied), which is less than the exponent 5 on the x term in the product. There is no y term in the potential factor. This is the same as there being a y term with an exponent 0 (because $y^0 = 1$). This is less than the exponent 3 on the y term in the product. The exponent on the z term in the potential factor is 2, which is less than the exponent 1 (implied) on the z term in the product. Therefore xz^2 is a factor of x^5y^3z.

xyz: The exponent on the x term in the potential factor is 1 (which is not displayed, but is implied), which is less than the exponent 5 on the x term in the product. The exponent on the y term in the potential factor is also 1 (implied), which is less than the exponent 3 on the y term in the product. The exponent on the z term in the potential factor is also 1 (implied), which is equal to the exponent 1 (implied) on the z term in the product. Therefore, xyz is a factor of x^5y^3z.

x^4y^4: The exponent on the x term in the potential factor is 4, which is less than the exponent 5 on the x term in the product. The exponent on the y term in the potential factor is also 4, which is greater than the exponent 3 on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 1 (implied) on the z term in the product. Therefore, x^4y^4 is not a factor of x^5y^3z (because of the y term).

ax^2y: The potential factor has an a term (with an implied exponent of 1), but there is no a term in the product. This is the same as there being an a term with an exponent of 0 (because $a^0 = 1$) in the product. The exponent on the a term is larger in the potential factor than on the a term in the product. The exponent on the x term in the potential factor is 2, which is less than the exponent 5 on the x term in the product. The exponent on the y term in the potential factor is 1 (implied), which is greater than the exponent 3 on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 1 (implied) on the z term in the product. Therefore ax^2y is not a factor of x^5y^3z.

In summary, xz^2 and xyz are factors of x^5y^3z, but x^4y^4 and ax^2y are not.

b) We will consider each potential factor separately.

x: The potential factor has a x term (with an implied exponent of 1), but there is no x term in the product. This is the same as there being an x term with an exponent of 0 (because $x^0 = 1$). The exponent on the x term is larger in the potential factor than on the x term in the product. There is no y term in the potential factor. This is the same as there being a y term with an exponent of 0 (because $y^0 = 1$). This is less than the exponent 1 (implied) on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 1 (implied) on the z term in the product. Therefore, x is not a factor of yz^2 (because of the x term).

yz: The exponent on the y term in the potential factor is 1 (implied), which is equal to the exponent 1 (implied) on the y term in the product. The exponent on the z term in the potential factor is also 1 (implied), which is less than the exponent 2 on the z term in the product. Therefore, yz is a factor of yz^2.

y^2z: The exponent on the y term in the potential factor is 2, which is greater than the exponent 1 (implied) on the y term in the product. The exponent on the z term in the potential factor is 2, which is equal to the exponent 2 on the z term in the product. Therefore, y^2z is not a factor of yz^2.

1: 1 is a factor of everything. Therefore 1 is a factor of yz^2.

In summary, yz and 1 are factors of yz^2, but x and y^2z are not.

c) We will consider each potential factor separately.

xy: The exponent on the x term in the potential factor is 1 (implied), which is equal to the exponent 1 (implied) on the x term in the product. The exponent on the y term in the potential factor is 1 (implied), which is equal to the exponent 1 (implied) on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent on the z term in the product which is 1 (implied). Therefore, xy is a factor of xyz.

xz: The exponent on the x term in the potential factor is 1 (implied), which is equal to the exponent 1 (implied) on the x term in the product. There is no y term in the potential factor. This is the same as there being a y term with an exponent of 0 (because $y^0 = 1$). This is less than the exponent 1 (implied) on the y term in the product. The exponent on the z term in the potential factor is 1 (implied), which is equal to the exponent 1 (implied) on the z term in the product. Therefore, xz is a factor of xyz.

x^2: The exponent on the x term in the potential factor is 2, which is greater than the exponent 1 (implied) on the x term in the product. There is no y term in the potential factor. This is the same as there being a y term with an exponent 0 (because $y^0 = 1$). This is less than the exponent 1 (implied) on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 1 (implied) on the z term in the product. Therefore, x^2 is not a factor of xyz (because of the x term).

q: The potential factor has a q term (with an implied exponent of 1), but there is no q term in the product. This is the same as there being a q term with an exponent 0 (because $q^0 = 1$). The exponent on the q term is larger in the potential factor than on the q term in the product. There is no x term in the potential factor. This is the same as there being a x term with an exponent of 0 (because $x^0 = 1$). This is less than the exponent 1 (implied) on the x term in the product. There is no y term in the potential factor. This is the same as there being a y term with an exponent of 0 (because $y^0 = 1$). This is less than the exponent on the y term in the product which is 1 (implied). There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 1 (implied) on the z term in the product. Therefore, q is not a factor of xyz (because of the q term).

In summary, xy and xz are factors of xyz, but x^2 and q are not.

d) We will consider each potential factor separately.

x^3y^2: The exponent on the x term in the potential factor is 3, which is greater than the exponent 2 on the x term in the product. The exponent on the y term in the potential factor is 2, which is less than the exponent 5 on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 3 on the z term in the product. Therefore, x^3y^2 is not a factor of $x^2y^5z^3$ (because of the x term).

$x^2y^3z^2$: The exponent on the x term in the potential factor is 2, which is equal to the exponent 2 on the x term in the product. The exponent on the y term in the potential factor is 3, which is less than the exponent 5 on the y term in the product. The exponent on the z term in the potential factor is 2, which is less than the exponent 3 on the z term in the product. Therefore, $x^2y^3z^2$ is a factor of $x^2y^5z^3$.

y^3x^2: In this potential factor, the variables are in a different order than in the product. This does not make a difference to the result. The exponent on the x term in the potential factor is 2, which is equal to the exponent 2 on the x term in the product. The exponent on the y term in the potential factor is 3, which is less than the exponent 5 on the y term in the product. There is no z term in the potential factor. This is the same as there being a z term with an exponent of 0 (because $z^0 = 1$). This is less than the exponent 3 on the z term in the product. Therefore, y^3x^2 is a factor of $x^2y^5z^3$.

$x^2y^5z^3$: The exponent on the x term in the potential factor is 2, which is equal to the exponent 2 on the x term in the product. The exponent on the y term in the potential factor is 5, which is equal to the exponent 5 on the y term in the product. The exponent on the z term in the potential factor is 3, which is equal to the exponent 3 on the z term in the product. Therefore, $x^2y^5z^3$ is a factor of $x^2y^5z^3$.

In summary, $x^2y^3z^2$, y^3x^2, and $x^2y^5z^3$ are factors of $x^2y^5z^3$, but x^3y^2 is not.

Solutions to problem set 6

a) The exponents on the x variable are 5, 3, and 2 respectively. The smallest exponent is 2. Therefore, x^2 can be factored out of all the terms.

   The exponents on the y variable are 2, 7, and 6. The smallest exponent is 2. Therefore, y^2 can be factored out of all the terms.

   The exponents on the z variable are 3, 2, and 10. The smallest exponent is 2. Therefore, z^2 can be factored out of all the terms.

   Putting these together, the largest shared factor is $x^2y^2z^2$.

b) The exponents on the x variable are 6, 9, and 5 respectively. The smallest exponent is 5. Therefore, x^5 can be factored out of all the terms.

   The exponents on the y variable are 3, 0, and 6. (There is no y variable in the second term.) The smallest exponent is 0. Therefore, no y variable can be factored out of all the terms ($y^0 = 1$).

   The exponents on the z variable are 2, 6, and 5. The smallest exponent is 2. Therefore, z^2 can be factored out of all the terms.

   Putting these together, the largest shared factor is x^5z^2.

c) The exponents on the x variable are 12, 8, and 0 respectively. (There is no x variable in the third term.) The smallest exponent is 0. Therefore, no x variable can be factored out of all the terms ($x^0 = 1$).

   The exponents on the y variable are 3, 0, and 9. (There is no y variable in the second term.) The smallest exponent is 0. Therefore, no y variable can be factored out of all the terms ($y^0 = 1$).

   The exponents on the z variable are 0, 8, and 4. (There is no z variable in the second term.) The smallest exponent is 0. Therefore, no z variable can be factored out of all the terms ($y^0 = 1$).

   There are no variables which are shared between all the terms. The largest shared factor is 1.

d) The exponents on the x variable are 9, 6, and 13 respectively. The smallest exponent is 6. Therefore, x^6 can be factored out of all the terms.

The exponents on the y variable are $5, 8$, and 8. The smallest exponent is 5. Therefore, y^5 can be factored out of all the terms.

The exponents on the z variable are $7, 10$, and 9. The smallest exponent is 7. Therefore, z^7 can be factored out of all the terms.

Putting these together, the largest shared factor is $x^6y^5z^7$.

Solutions to problem set 7

a) From the previous problem set, we know the largest variable factor that is shared by all the coefficients is $x^2y^2z^2$.

Multiply this expression by $1 = \frac{x^2y^2z^2}{x^2y^2z^2}$ and re-arrange the expression:

$$\frac{x^2y^2z^2}{x^2y^2z^2}(9x^5y^2z^3 + 3x^3y^7z^2 - 2x^2y^6z^10) = x^2y^2z^2\left(\frac{9x^5y^2z^3}{x^2y^2z^2} + \frac{3x^3y^7z^2}{x^2y^2z^2} - \frac{2x^2y^6z^{10}}{x^2y^2z^2}\right)$$

Next, simplify the fractions:

$$x^2y^2z^2(9x^3y^0z^1 + 3x^1y^5z^0 - 2x^0y^4z^8)$$

Finally, clean up the expression:

$$x^2y^2z^2(9x^3z + 3xy^5 - 2y^4z^8)$$

b) From the previous problem set, we know the largest variable factor that is shared by all the coefficients is x^5z^2.

Multiply this expression by $1 = \frac{x^5z^2}{x^5z^2}$ and re-arrange the expression:

$$\frac{x^5z^2}{x^5z^2}(5x^6y^3z^2 - 6x^9z^6 + 2x^5y^2) = x^5z^2\left(\frac{5x^6y^3z^2}{x^5z^2} - \frac{6x^9z^6}{x^5z^2} + \frac{2x^5y^2z^5}{x^5z^2}\right)$$

Next, simplify the fractions:

$$x^5z^2(5x^1y^3z^0 - 6x^4z^4 + 2x^0y^2z^3)$$

Finally, clean up the expression:

$$x^5z^2(5xy^3 - 6x^4z^4 + 2y^2z^3)$$

c) From the previous problem set, we know the largest variable factor that is shared by all the coefficients is 1.

You may notice immediately that factoring out a 1 will not change an expression. However, we can still go through the process of factoring to confirm this.

Multiply this expression by $1 = \frac{1}{1}$ and re-arrange the expression:

$$\frac{1}{1}(10x^12y^3 + 5x^8z^8 + 6y^9z^4) = 1\left(\frac{10x^12y^3}{1} + \frac{5x^8z^8}{1} + \frac{6y^9z^4}{1}\right)$$

We can then simplify this expression:

$$1(10x^12y^3 + 5x^8z^8 + 6y^9z^4)$$

Multiplying 1 by anything does not change the expression, so this is equivalent to:

$$10x^12y^3 + 5x^8z^8 + 6y^9z^4$$

d) From the previous problem set, we know the largest variable factor that is shared by all the coefficients is $x^6y^5z^7$.

Multiply this expression by $1 = \frac{x^6y^5z^7}{x^6y^5z^7}$ and re-arrange the expression:

$$\frac{x^6y^5z^7}{x^6y^5z^7}(3x^9y^5z^7 - 2x^6y^8z^6 - 9x^{13}y^8z^{10}) = x^6y^5z^7\left(\frac{3x^9y^5z^7}{x^6y^5z^7} - \frac{2x^6y^8z^{10}}{x^6y^5z^7} - \frac{9x^{13}y^8z^9}{x^6y^5z^7}\right)$$

Next, simplify the fractions:

$$x^6y^5z^7(3x^3y^0z^0 - 2x^0y^3z^5 - 9x^7y^3z^2)$$

Finally, clean up the expression:

$$x^6y^5z^7(3x^3 - 2y^3z^5 - 9x^7y^3z^2)$$

Solutions to problem set 8

a) The coefficients are 4, -8, and 6. The positive integer factors of 4 are 1, 2, and 4. The positive integer factors of -8 are 1, 2, 4, and 8. The positive integer factors of 6 are 1, 2, 3, and 6. The largest shared factor is 2.

The exponents on the x variable are 5, 3, and 4 respectively. The smallest exponent is 3. Therefore, x^3 can be factored out of all the terms.

The exponents on the y variable are 3, 5, and 3 respectively. The smallest exponent is 3. Therefore, y^3 can be factored out of all the terms.

The exponents on the z variable are 6, 2, and 7 respectively. The smallest exponent is 2. Therefore, z^2 can be factored out of all the terms.

Putting these together, the greatest common factor is $2x^3y^3z^2$.

Multiply the expression by $1 = \frac{2x^3y^3z^2}{2x^3y^3z^2}$, then re-distribute the fraction.

$$\frac{2x^3y^3z^2}{2x^3y^3z^2}(4x^5y^3z^6 - 8x^3y^5z^2 + 6x^4y^3z^7) = 2x^3y^3z^2\left(\frac{4x^5y^3z^6}{2x^3y^3z^2} - \frac{8x^3y^5z^2}{2x^3y^3z^2} + \frac{6x^4y^3z^7}{2x^3y^3z^2}\right)$$

Next, simplify the fractions:

$$2x^3y^3z^2(2x^2y^0z^4 - 4x^0y^2z^0 + 3x^1y^0z^5)$$

Finally, clean up the expression:

$$2x^3y^3z^2(2x^2z^4 - 4y^2 + 3xz^5)$$

b) The coefficients are 5 and 10. The positive integer factors of 5 are 1 and 5. The positive integer factors of 10 are 1, 2, 5, and 10. The largest shared factor is 5.

The exponents on the x variable are 1 and 2 respectively. The smallest exponent is 1. Therefore, $x^1 = x$ can be factored out of all the terms.

The exponents on the y variable are both 1. The smallest exponent is 1. Therefore, $y^1 = y$ can be factored out of all the terms.

The exponents on the z variable are 3 and 5 respectively. The smallest exponent is 3. Therefore, z^3 can be factored out of all the terms.

Putting these together, the greatest common factor is $5x^1y^1z^3 = 5xyz^3$.

Multiply the expression by $1 = \frac{5xyz^3}{5xyz^3}$, then re-distribute the fraction.

$$\frac{5xyz^3}{5xyz^3}(5xyz^3 + 10x^2yz^5) = 5xyz^3\left(\frac{5xyz^3}{5xyz^3} + \frac{10x^2yz^5}{5xyz^3}\right)$$

Next, simplify the fractions:

$$5xyz^3(1x^0y^0z^0 + 2x^1y^0z^2)$$

Finally, clean up the expression:

$$5xyz^3(1 + 2xz^2)$$

c) The coefficients are 8 and -12. The positive integer factors of 8 are 1, 2, 4, and 8. The positive integer factors of -12 are 1, 2, 3, 4, 6, and 12. The largest shared factor is 4.

The exponents on the x variable are 3 and 2 respectively. The smallest exponent is 2. Therefore, x^2 can be factored out of all the terms.

The exponents on the y variable are 2 and 0 respectively. The smallest exponent is 0. Therefore, $y^0 = 1$ can be factored out of all the terms.

Putting these together, the greatest common factor is $4x^2y^0 = 4x^2$.

Multiply the expression by $1 = \frac{4x^2}{4x^2}$, then re-distribute the fraction.

$$\frac{4x^2}{4x^2}(8x^3y^2 - 12x^2) = 4x^2\left(\frac{8x^3y^2}{4x^2} - \frac{12x^2}{4x^2}\right)$$

Next, simplify the fractions:

$$4x^2(2x^1y^2 - 3x^0)$$

Finally, clean up the expression:

$$4x^2(2xy^2 - 3)$$

d) The coefficients are 7 and 8. The positive integer factors of 7 are 1 and 7. The positive integer factors of 8 are 1, 2, 4, and 8. The largest shared factor is 1.

The exponents on the x variable are 1 and 0 respectively. The smallest exponent is 0. Therefore, $x^0 = 1$ can be factored out of all the terms.

The exponents on the y variable are 0 and 2 respectively. The smallest exponent is 0. Therefore, $y^0 = 1$ can be factored out of all the terms.

Putting these together, the greatest common factor is $1x^0y^0 = 1$.

The only shared factor is 1, which means that no common factor can be factored out of this expression.