The Mathematical Path

Introduction to Fractions

Fractions are a part of a whole. We use the idea of fractions in our everyday lives when telling time, cooking, and sharing things with friends. Fractions are also a fundamental part of mathematics that will show up all the time in your future mathematical work.

This module will introduce the fundamental ideas of fractions. The ideas presented here will be expanded upon in subsequent modules where we begin working with factions. This module assumes you're familiar with:

- Counting

This module is a part of The Mathematical Path, which introduces mathematical ideas step-by-step to ensure that everyone's able to follow along. The latest version of the file can be obtained from `www.mathematicalpath.com/` along with other supporting materials.

Remember that mathematics builds upon itself, so if you're struggling with this material, it could be because you didn't quite understand something earlier on. Don't be afraid to back up and review some earlier mathematical concepts to get a stronger foundation and then come back to this module when you're ready. Visit The Mathematical Path website to trace back until you reach material you're comfortable with and then work your way back up to this module.

Be sure to visit `www.mathematicalpath.com/support/` to learn how you can support The Mathematical Path so we can create more modules!

When we first start working with numbers, we naturally use whole numbers, counting objects 1, 2, 3, 4, and so on. But at some point you probably tried to divide an even number into two equal groups and realized it wouldn't work. One solution is to divide one of the objects into smaller pieces which are less than a whole object. Fractions are one way of talking about numbers that are part of a whole rather than a whole number.

There are two ways of thinking about fractions: in terms of size or in terms of sets. An example of when size matters with fraction is if you have a cake; you divide the whole cake up into equal size pieces. An example of working with fractions on a set is if you have a bag of candies; each piece of candy counts as one piece, even if they're not exactly the same size.

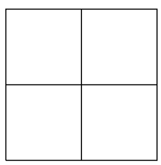
Fractions are written as one number over the other with either a horizontal or diagonal line between them. This line is called the "fraction bar". The number on the top of the fraction is called the "numerator". The number on the bottom of the fraction is called the "denominator".

horizontal fraction bar: $\frac{numerator}{denominator}$ diagonal fraction bar: $^{numerator}/_{denominator}$

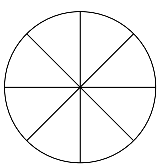
Finding the denominator

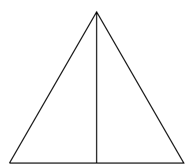
The number on the bottom of the fraction is called the denominator. This is the total number of pieces that make up a whole.

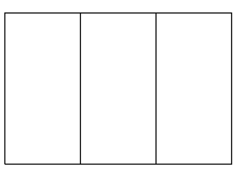
Problem set 1 (finding the denominator for sizes):
For each shape, find the denominator for the fraction by counting the number of pieces in the shape.

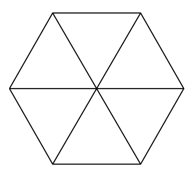
Sample problem: 

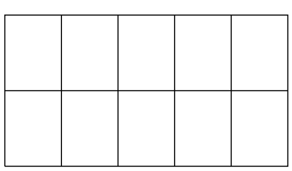
Sample solution: The denominator is 4, because this square is divided into 4 pieces. As a fraction, this would be written as $\frac{}{4}$.

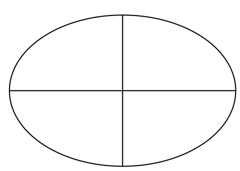
a) 

b) 

c) 

d) 

e) 

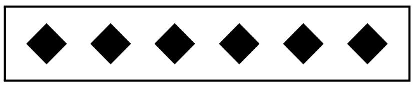
f) 

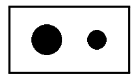
Problem set 2 (finding the denominator for sets):

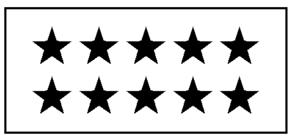
For each set of symbols, find the denominator for the fraction by counting the number of symbols in the set.

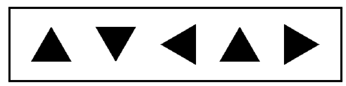
Sample problem: 

Sample solution: The denominator is 3, because there are 3 stars in this set. As a fraction, this would be written as $\frac{\;}{3}$.

a) 

b) 

c) 

d) 

Denominator names

When speaking of fractions in English, we need words to describe the fractions. Different denominators have different words.

If the denominator is 2, then one piece is call a "half". Multiple pieces are described as "halves".

If the denominator is 3, then one piece is called a "third". Multiple pieces are described as "thirds'.

If the denominator is 4, then one piece is called a "quarter". Multiple pieces are described as "quarters".

If the denominator is 5, then one piece is called a "fifth". Multiple pieces are described as "fifths".

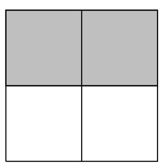
Beyond this, one piece is generally called the number of pieces with "-th" added as a suffix. For example, if the denominator is 26, then one piece is called a "twenty-sixth" and multiple pieces are described as "twenty-sixths". The exception to this rule is if the last digit of the denominator is either a 3 or a 5. In this case, we still use "-third" or "-fifth". For example, if the denominator is 25, then one piece is called a "twenty-fifth".

Finding the numerator

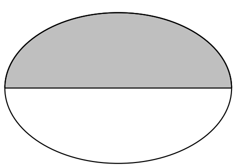
The number on the top of the fraction is called the numerator. This is the number of pieces that meet the condition we are talking about. For example, it could be the fraction of candies you get to eat, the fraction of the pizza that has pepperoni on it, or the fraction of people in a room who are not wearing hats.

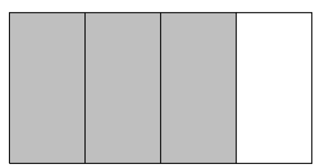
It is important to focus only on the specific condition and not get distracted by other characteristics. For example, the fraction of candies you get to eat is not impacted by the flavour of the candies, the amount of the pizza that has pepperoni is not impacted by the presence or absence of pineapple, and the number of people who are not wearing hats is not impacted by the height of those people.

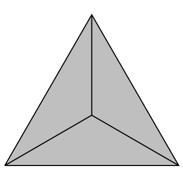
Problem set 3 (finding the numerator for sizes):
For each shape, complete the fraction by counting the number of pieces of the shape that are shaded.

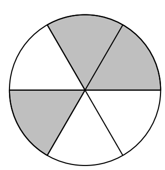
Sample problem:  $\overline{4}$

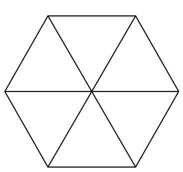
Sample solution: We know that the denominator is 4. We count and find that 2 of these pieces are shaded. Therefore, the numerator is 2. As a fraction, this would be written as $\frac{2}{4}$.

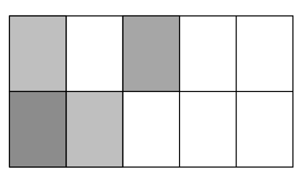
a)  $\overline{2}$

b)  $\overline{4}$

c)  $\overline{3}$

d)  $\overline{6}$

e)  $\overline{6}$

f)  $\overline{10}$

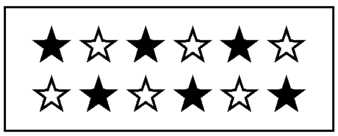
Problem set 4 (finding the numerator for sets):

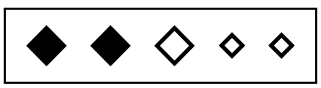
For each set of symbols, complete the fraction by counting the number of elements of the set that are black.

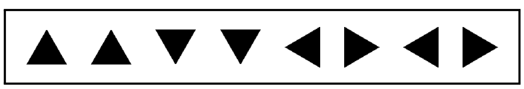
Sample problem:  $\frac{\quad}{3}$

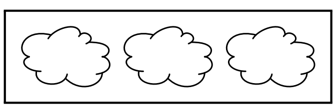
Sample solution: We know that the denominator is 3. We count and find that 2 of these pieces are black. Therefore, the numerator is 2. As a fraction, this would be written as $\frac{2}{3}$.

a)  $\frac{\quad}{4}$

b)  $\frac{\quad}{12}$

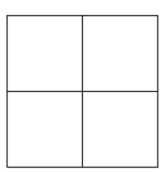
c)  $\frac{\quad}{5}$

d)  $\frac{\quad}{8}$

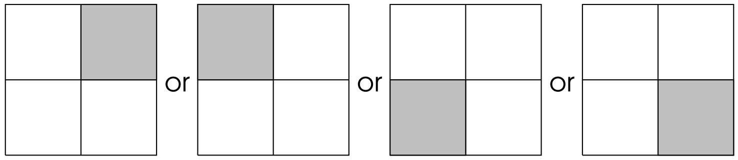
e)  $\frac{\quad}{3}$

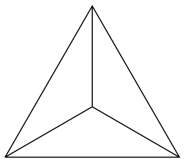
Problem set 5 (illustrating fractions):

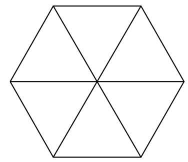
For each shape or set, shade the appropriate number of pieces to illustrate the fraction. Be sure to check that the number of pieces and the denominator match–if they don't, make a note that you cannot shade the pieces appropriately.

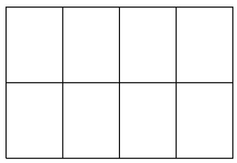
Sample problem: $\frac{1}{4}$ 

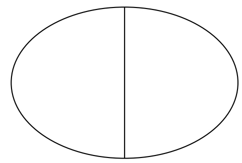
Sample solution: The denominator is 4, which matches the number pieces. Since the numerator is 1, we should shade 1 piece. It does not matter which piece is selected so any of these are correct:

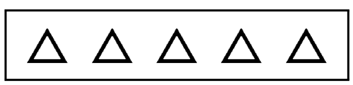


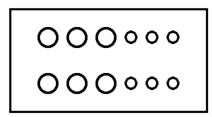
a) $\frac{1}{3}$ 

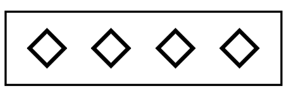
b) $\frac{5}{6}$ 

c) $\frac{3}{8}$ 

d) $\frac{2}{2}$ 

e) $\frac{3}{5}$ 

f) $\frac{6}{12}$ 

g) $\frac{0}{4}$ 

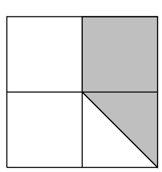
h) $\frac{4}{5}$ 

Equal size parts

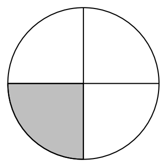
When working with fractions in terms of size, it's important that each of the part of the whole has the same size. Imagine a friend offers to give you half their cookie. If they only gave you a tiny piece and kept most of it to themself, you'd probably be surprised and disappointed.

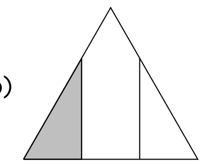
Problem set 6:

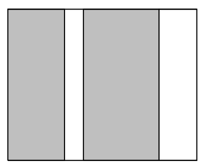
For each shape, if the pieces are equal sizes, write the fraction of the shape that is shaded. If the pieces are not equal sizes, note that you cannot tell the fraction just from looking at the shape.

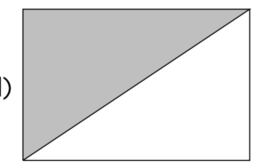
Sample problem: 

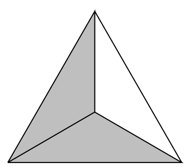
Sample solution: The 2 pieces in the lower right corner are not the same size as the other 3. Therefore, we cannot immediately tell the fraction that is shaded from the picture.

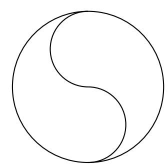
a) 

b) 

c) 

d) 

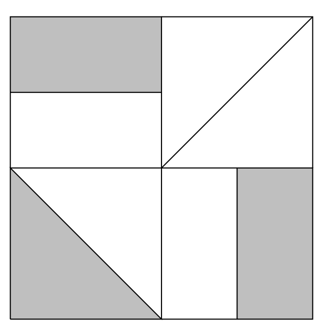
e) 

f) 

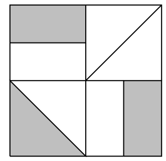
Conversely, when working with fractions in terms of sets, each item counts as one item, no matter what its size.

Different shapes, same sizes

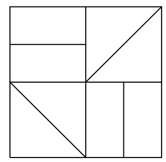
Remember that when we talk about fractions in terms of the size, it's specifically the size that matters, not the shape. Consider the following shape, can we tell what fraction is shaded from the picture?



The 8 pieces of this square are different shapes. However, a careful examination shows that they are all the same size! You can see this by first focusing on the lines through the center both vertically and horizontally, dividing the square into 4 pieces.



Then each of the squares is divided evenly in half.



Since every piece was always evenly divided, we know that each piece is the same size, even without having to figure out the actual area! There are 8 pieces total, so the dominator is 8. There are 3 pieces shaded, so the numerator is 3. Therefore, fraction is $\frac{3}{8}$.

Key Points

- Fractions are parts of a whole, which can be based on size or sets.
- When talking about fractions in terms of sets, each item counts as 1.
- When talking about fractions in terms of size, the pieces must be equal sizes.
- The numerator (top number) of the fraction is the number of parts that meet the condition in question.
- The denominator (bottom number) of the fraction is the total number of parts.

Next Steps

If you've read through this module, make sure you do all the exercises and check the solutions in the next section. If you got any answers wrong, go over the solutions and try to understand why you got a different answer.

If you are still feeling confused or don't understand something, please let us know. We're always willing to revisit and improve modules to ensure that everyone is able to follow along. It's not your fault for not understanding something, it's our fault it wasn't presented clearly enough! The best way for us to improve in order to help you and the next person who comes along is for you to tell us what didn't make sense. Similarly, if there's something you thought should be covered in this module but wasn't, be sure to let us know.

If you're feeling confident in this material, check out the www.mathematicalpath.com/ to find more modules that build on this one. If there's a module you want but doesn't exist yet, contact us to make a request and help us prioritize our efforts. We hope to see you soon as you continue down the Mathematical Path.

Solutions

Solutions to problem set 1

a) The denominator is 8, because this square is divided into 8 pieces. As a fraction, this would be written as $\frac{}{8}$.

b) The denominator is 2, because this triangle is divided into 2 pieces. As a fraction, this would be written as $\frac{}{2}$.

c) The denominator is 3, because this rectangle is divided into 3 pieces. As a fraction, this would be written as $\frac{}{3}$.

d) The denominator is 6, because this hexagon is divided into 6 pieces. As a fraction, this would be written as $\frac{}{6}$.

e) The denominator is 10, because this rectangle is divided into 10 pieces. As a fraction, this would be written as $\frac{}{10}$.

f) The denominator is 4, because this oval is divided into 4 pieces. As a fraction, this would be written as $\frac{}{4}$.

Solutions to problem set 2

a) The denominator is 6, because there are 6 diamonds in the set. As a fraction, this would be written as $\frac{}{6}$.

b) The denominator is 2, because there are 2 circles in the set. It does not matter that the circles are different sizes. As a fraction, this would be written as $\frac{}{2}$.

c) The denominator is 10, because there are 10 stars in the set. As a fraction, this would be written as $\frac{}{10}$.

d) The denominator is 5, because there are 5 triangles in the set. It does not matter that the triangles point in different directions. As a fraction, this would be written as $\frac{}{5}$.

Solutions to problem set 3

a) We know that the denominator is 2. We count and find that 1 of these pieces are shaded. Therefore, the numerator is 1. As a fraction, this would be written as $\frac{1}{2}$.

b) We know that the denominator is 4. We count and find that 3 of these pieces are shaded. Therefore, the numerator is 3. As a fraction, this would be written as $\frac{3}{4}$.

c) We know that the denominator is 3. We count and find that all 3 of these pieces are shaded. Therefore, the numerator is 3. As a fraction, this would be written as $\frac{3}{3}$.

d) We know that the denominator is 6. We count and find that 3 of these pieces are shaded. Therefore, the numerator is 3. It does not matter that the shaded pieces are not grouped together. As a fraction, this would be written as $\frac{3}{6}$.

e) We know that the denominator is 6. We count and find that none of these pieces are shaded. Therefore, the numerator is 0. As a fraction, this would be written as $\frac{0}{6}$.

f) We know that the denominator is 10. We count and find that 4 of these pieces are shaded. Therefore, the numerator is 4. It does not matter that some are shaded lighter or darker as the condition to count was just "shaded". As a fraction, this would be written as $\frac{4}{10}$.

Solutions to problem set 4

a) We know that the denominator is 4. We count and find that 3 of these pieces are black. Therefore, the numerator is 3. As a fraction, this would be written as $\frac{3}{4}$.

b) We know that the denominator is 12. We count and find that 6 of these pieces are shaded. Therefore, the numerator is 6. It does not matter that the black stars are spread out. As a fraction, this would be written as $\frac{6}{12}$.

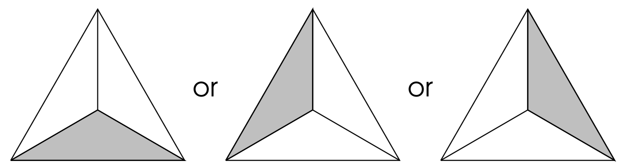
c) We know that the denominator is 5. We count and find that 2 of these pieces are black. Therefore, the numerator is 2. It does not matter that the diamonds are different sizes. As a fraction, this would be written as $\frac{2}{5}$.

d) We know that the denominator is 8. We count and find that all 8 of these pieces are black. Therefore, the numerator is 8. It does not matter that the triangles are pointing in different directions. As a fraction, this would be written as $\frac{8}{8}$.

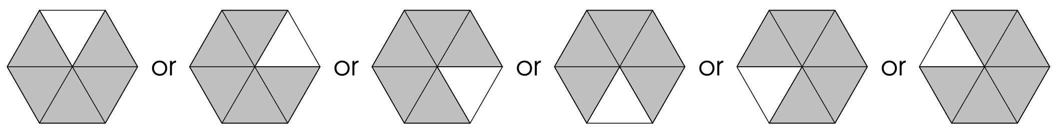
e) We know that the denominator is 3. We count and find that none of these pieces are black. Therefore, the numerator is 0. As a fraction, this would be written as $\frac{0}{3}$.

Solutions to problem set 5

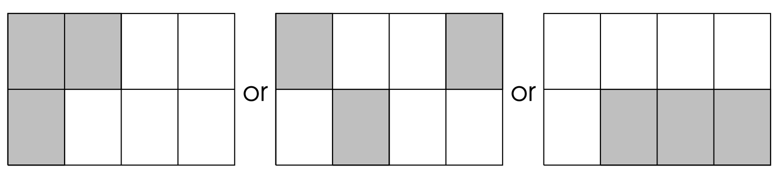
a) The denominator is 3, which matches the number pieces. Since the numerator is 1, we should shade 1 piece. It does not matter which piece is selected so any of these are correct:



b) The denominator is 6, which matches the number pieces. Since the numerator is 5, we should shade 5 pieces. It does not matter which pieces are selected so any of these are correct:

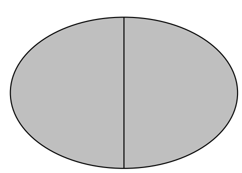


c) The denominator is 8, which matches the number pieces. Since the numerator is 3, we should shade 3 pieces. There are actually 56 different ways to shade 3 of the 8 pieces, but here are a few examples:



or. . . .

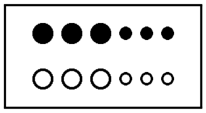
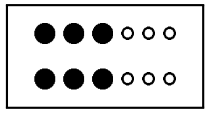
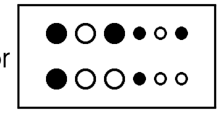
d) The denominator is 2, which matches the number pieces. Since the numerator is 2, we should shade both pieces. There's only one way to do this:



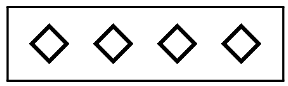
e) The denominator is 5, which matches the number pieces. Since the numerator is 3, we should shade 3 pieces. There are actually 10 different ways to shade 3 of the 5 pieces, but here are a few examples:

▲ ▲ ▲ △ △ or ▲ △ ▲ △ ▲ or △ ▲ △ ▲ ▲ or. . . .

f) The denominator is 12, which matches the number pieces. Since the numerator is 6, we should shade 6 pieces. It does not matter which 6 pieces, despite the differences in size. There are actually 924 different ways to shade 6 of the 12 pieces, but here are a few examples:

 or  or  or . . . .

g) The denominator is 4, which matches the number pieces. Since the numerator is 0, we should not shade any pieces. There's only one way to do this:



h) The denominator is 5, which does not match the number pieces. Therefore, we cannot shade a fraction of $\frac{4}{5}$ in this set.

Solutions to problem set 6

a) The 4 pieces of this circle are all the same size. Therefore, we can evaluate the fraction. There are 4 pieces total, so the denominator is 4. There are no pieces shaded, so the numerator is 0. The fraction is $\frac{0}{4}$.

b) Even though the lower edge is divided evenly into 3, the 3 pieces overall are not the same size. Therefore, we cannot immediately tell the fraction that is shaded from the picture.

c) The 4 pieces of the rectangle are not the same size. Therefore, we cannot immediately tell the fraction that is shaded from the picture.

d) The 2 pieces of this rectangle are the same size. Therefore, we can evaluate the fraction. There are 2 pieces total, so the denominator is 2. There is 1 piece shaded, so the numerator is 1. The fraction is $\frac{1}{2}$.

e) The 3 pieces of this triangle are all the same size. Therefore, we can evaluate the fraction. There are 3 pieces total, so the denominator is 3. There are 2 piece shaded, so the numerator is 2. The fraction is $\frac{2}{3}$.

f) The 2 pieces of this circle are the same size. Therefore, we can evaluate the fraction. There are 2 pieces total, so the denominator is 2. Neither of the pieces are shaded, so the numerator is 0. The fraction is $\frac{0}{2}$.